Mixing Gaussian Models to Price CMS Derivatives

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Abstract

In this article, we propose a simple interest rate model, which can well accommodate swaption smiles, while recovering market prices of CMS swap spreads. The model is based on a (possibly multi-factor) Gaussian short rate model coupled with parameter uncertainty. Examples of calibration to real market data will be presented as well as the pricing of some typical CMS-based derivatives.

1 Introduction

Swap rate dependent derivatives have become increasingly popular in the interest rate market. Typical examples are the CMS spreads and CMS spread options, whose payoffs are based on the difference between long and short maturities swap rates. Such payoffs are usually embedded in swaps, which may be cancelled due to Bermudan-style callability features or to TARN-style conditions.

To be correctly priced, such derivatives need a model that incorporates at once the market swaption volatilities of all the relevant swap rates (both at-the-money and away-from-the-money) consistently with a realistic form for their correlation term structure.

Swaption volatilities are usually quoted in the market by means of the SABR functional form, see Hagan et al. (2002), implicitly assuming a different (and independent) stochastic volatility model for each different swap rate. Motivating this practice on the basis of a proper market model is still an issue to be addressed. On the other hand, one can simply regard the swaption smile as a market input to which to calibrate one's favorite interest rate model. In this case, an effective and rather straightforward solution is obtained by coupling a short-rate model with parameter uncertainty in the spirit of Brigo, Mercurio and Rapisarda (2004). This approach, therefore, naturally leads to consider mixtures of short rate models.

In this article, we model the short rate process by means of a mixture of Gaussian models, possibly multi-factor in each of its components. The model can be calibrated to at-the-money swaption volatilities, swaption volatility skews, implied CMS convexity adjustments and swap correlation term structures. Examples of calibration to real market data will be shown. Pricing of relevant derivatives will be presented too.

The interest rate model we propose may appear out of date in an era where LIBOR models seem to be the obligatory path to follow. However, besides their undisputable advantages, LIBOR models also possess a number of drawbacks, especially concerning the rigidity of the associated time structure. In this respect, one may want to implement a simpler solution that is anyway able to accommodate the relevant market data, while being also relatively easy to implement. The short rate model with uncertain parameters we propose in this article is clearly an attempt to address this issue.

2 Mixing Short-Rate Gaussian Models

Consider an interest-rate model where the instantaneous short-rate process is given by the sum of q correlated Gaussian factors plus a deterministic function that is properly chosen so as to exactly fit the current term structure of discount factors $T \rightarrow P^{\mathrm{MKT}}(0,T)$ observed in the market. Assume that the dynamics of the instantaneous short-rate process under the risk-adjusted measure Q is given by

$$r(t) = \varphi(t) + \sum_{k=1}^{q} x_k(t) \tag{1}$$

where $\varphi(t)$ is the deterministic function to be used for exact calibration of the current zero coupon curve, and the processes $x_k(t)$, with $x_k(0) = 0$, satisfy

$$dx_k(t) = -a_k x_k(t)\, dt + \sigma_k\, dW_k(t) \tag{2}$$

where $W = (W_1, \dots, W_q)$ is a q-dimensional Brownian motion with instantaneous correlations $\rho_{kj} \in [-1, 1]$ as from

$$dW_k(t)\, dW_j(t) = \rho_{kj}\, dt, \tag{3}$$

and the speeds of mean reversion a_k and the volatility coefficients σ_k are positive constants.

Closed formulas for zero-coupon bonds and plain vanilla options under (1) can be straightforwardly obtained by standard techniques. In the one- and two-factor cases, they can be found, for instance, in Brigo and Mercurio (2001).

The model parameters a_k, σ_k and ρ_{kj} are typically calibrated either to caplet or swaption volatilities. If model (1) has more than one factor (*i.e.* $q > 1$), one may also calibrate some kind of market implied correlation between relevant rates, as we will see in the calibration section below. However, short-rate Gaussian models are characterized, as far as both caplets and swaptions are concerned, by an endogenous volatility skew, which is monotonically decreasing as the option strike increases, while market volatilities often show an increasing behavior for large strike values (hockey stick shape).

A natural way to obtain a more general implied volatility surface is to introduce an uncertain parameter model (UPM) with N components, each component $i \in \{1, \ldots, N\}$ describing a Gaussian model with q_i factors. We thus assume that the dynamics of the instantaneous short-rate process under the risk-adjusted measure Q is given by

$$r(t) = \varphi_I(t) + \sum_{k=1}^{q_I} x_k(t; I) \tag{4}$$

with

$$dx_k(t; I) = -a_k^I x_k(t; I)\, dt + \sigma_k^I\, dW_k(t),$$

where I is a discrete random variable, independent of the Brownian motion W, taking values in the set $\{1, \ldots, k\}$ with probabilities $\lambda_i := Q(I = i) > 0$, and where a_k^i and $\sigma_k^i$are positive constants for each i, j, k and φ_i are deterministic functions.[1] The random value of I is drawn at an infinitesimal instant after time 0, reflecting the initial uncertainty on which scenario will occur in the (near) future.

In this article, the UPM models (4) will be denoted following the rule G$q_1 q_2 \ldots$ where the qs are the number of factors in each component of the mixture. For example, the standard two-factor Hull and White (1994b) model will be denoted by G2, while a mixture of three one-factor Hull and White (1994a) models by G111.

3 Modelling the Function φ_I

The function $\varphi_I(t)$ can be modelled in a number of different ways, according to the dependence of φ on the model parameters that is assumed. We distinguish the following cases:

1. $\varphi_I(t)$ is the same for each component, namely $\varphi_i = \varphi_j = \varphi$ for each i, j. The function φ is then found by differentiating, with respect to T, both sides of

$$P^{\mathrm{MKT}}(0, T) = e^{-\int_0^T \varphi(t)\, dt} \sum_{i=1}^{N} \lambda_i E\left[e^{-\int_0^T \sum_{k=1}^{q_i} x_k(t; i)\, dt}\right],$$

   where E denotes expectation under the risk neutral measure;

2. $\varphi(t)$ changes with the component, in that each component i is associated with a different function $\varphi_i(t)$. We then have the following two subcases:

[1]We are here restricting ourselves to the case where correlations ρ_{kj} are scenario independent. In general, one can also consider uncertain correlations ρ_{kj}^I and assume that I is independent of the standard Brownian motion Z coming form the Cholesky decomposition W=CZ, where $R = CC'$ is the correlation matrix of W.

(a) Each $\varphi_i(t)$ is chosen so as to exactly fit the term structure $T \to P^{\mathrm{MKT}}(0,T)$ for each component i, independently from the others. Functions φ_i are then found, for each i, by differentiating, with respect to T, both sides of:

$$P^{\mathrm{MKT}}(0,T) = e^{-\int_0^T \varphi_i(t)\,dt} E\Big[e^{-\int_0^T \sum_{k=1}^{q_i} x_k(t;i)\,dt}\Big].$$

(b) Functions $\varphi_i(t)$ are chosen so as to exactly fit the term structure $T \to P^{\mathrm{MKT}}(0,T)$ globally (and not for each single component). In this case, the equality

$$P^{\mathrm{MKT}}(0,T) = \sum_{i=1}^{N} \lambda_i\, e^{-\int_0^T \varphi_i(t)\,dt} E\Big[e^{-\int_0^T \sum_{k=1}^{q_i} x_k(t;i)\,dt}\Big],$$

which holds for each T, defines an implicit relation between functions φ_i in that one can freely choose $N-1$ of them and define the remaining one accordingly.

Case (2b) is the most general one, including both (1) and (2a) as particular subcases. In this article, we will stick to (2a), since it allows us to perform separate calibrations to the current term structure of discount factors and calculate model prices by simply mixing the (yield curve consistent) prices obtained under each scenario. In fact, the price π^H of a given derivative H, is given by

$$\pi^H = \sum_{i=1}^{N} \lambda_i \pi_i^H, \tag{5}$$

where π_i^H is the derivative's price under the short rate model

$$r_i(t) = \varphi_i(t) + \sum_{k=1}^{q_i} x_k(t;i).$$

We refer to Brigo, Mercurio and Rapisarda (2004) for related details.

4 Calibration to Market Data

An accurate pricing of CMS derivatives relies both on swaption volatility smiles and swap rate correlations. Our strategy is to build a UPM model where the first component incorporates the correlation effects, while the other component(s) the smile effects.

The first component must have at least two factors to cope with correlations. On the other hand Gaussian models with more than two factors lead to too complex expressions for swaption pricing to include them in an effective calibration. We thus choose a two-factor Gaussian model for the first component. The remaining ones are to be used to better accommodate the smile surface, a task that can already be achieved with one-factor models. We have investigated several cases of calibration to market data. They have led us to the conviction that the G211 model is a good candidate, in that a mixture of one two-factor model and two one-factor models seems to reach a good tradeoff between model complexity and quality of calibration.

4.1 Volatility Smiles and Correlations

Swaption smiles are quoted by the market either directly, as implied volatilities for away-from-the-money swaptions, or indirectly in terms of CMS swap spreads. Swap rate correlations are not directly quoted by the market, but their values can possibly be deduced from the available prices of CMS derivatives such as CMS spread options, as soon as CMS convexity adjustments are known (whose value, in turn, depends on the smile effects). In order to extract from the market the needed information, we employ the following procedure.

1. Since swaption volatility smiles are typically constructed according to the SABR functional form, we can calculate any missing quote by using this form with parameters calibrated to the available market volatilities.

2. The market (implied) data for CMS convexity adjustments can be obtained from the quotes of CMS swaps spreads by means of a static replication argument, once swaption volatilities are known for all possible strikes,

3. Once CMS convexity adjustments are obtained for the relevant maturities and tenors, it is possible to infer from the prices of CMS spread options the implied value of the related swap rate correlations by means of a two-asset Black-like model.

In our calibration procedure, we will also minimize the difference between market implied data, for swap rate correlations and CMS convexity adjustments, and the corresponding values obtained with a Monte Carlo simulation, keeping fixed the simulation seed while varying the model parameters.

4.2 Calibration Procedure

Our calibration procedure of the G211 model is based on an extension of the local optimization algorithm by Levenberg and Marquardt, see Lourakis (2005). Our objective function incorporates at-the-money swaption volatilities, swaption volatility skews, implied CMS convexity adjustments and swap rate correlation term structures.

We experience that the local optimization employed to calibrate the G211 model is independent of the initial guess, but it can be quite time consuming, due to nested Monte Carlo, if the initial guess is not properly fixed. A quick recipe consists first in calibrating a G2 model only to at-the-money swaption volatilities and swap rate correlations, then, in using the G2 calibrated parameters as initial guess for the first component of the G211 model, while the initial guess for the parameters of the other two components can be fixed arbitrarily.

Note that the calibration of the G2 model only to at-the-money swaption volatilities may depend on the initial guess due to the presence of many equally sized minima for the objective function, while calibration of the G2 model to both at-the-money volatilities and swap rate correlations turns out to be quite insensitive to the initial guess.

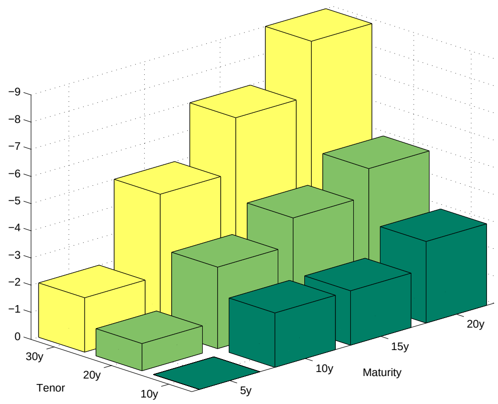

Figure 1: Absolute differences in basis points between market CMS swap spreads and those induced by the G211 model.

4.3 Calibration Results

Our examples of calibration are based on Euro data as of 28 September 2005, which we report in Tables 8, 1, 2, 3 and 4. We list swaption volatilities for different strikes, CMS-swap spreads for different swap-rate tenors and swap-rate correlations for different expiries.

The G211 model is able to fit the considered volatility data within errors of few tenths of percentage points. Swap-rate correlations and CMS-swap spreads are also recovered quite well. We report the obtained numerical values in Tables 5, 6 and 7. We also also provide graphical representations in Figures 1 and 2.

Remark 4.1. *Adding two one-factor components to the G2 model does not lower significantly the calibration error as far as at-the-money swaption volatilities are concerned, but it allows to better fit away-from-the-money data, as formerly stated when selecting a candidate Gaussian model.*

Finally, the swaption smiles induced by the G211 model, for a few relevant expiries and tenors, are shown in Table 9 and Figure 3, where they are reported as differences with respect to the corresponding at-the-money values. There we can clearly see that, contrary to the basic Gaussian short rate models, the G211 implied volatilities have a quite realistic behavior, with values that increase for large strikes.

5 Pricing CMS Derivatives

In this section, we tackle the pricing issue of a popular exotic CMS derivative, which is sensitive both to the swaption smile and to swap rate correlations. To this end, we will use the G211 model and compare the resulting prices with those produced by the G2 model. This comparison, within the limit of using two different models, serves the purpose of highlighting the possible price differences induced by a calibration that misses

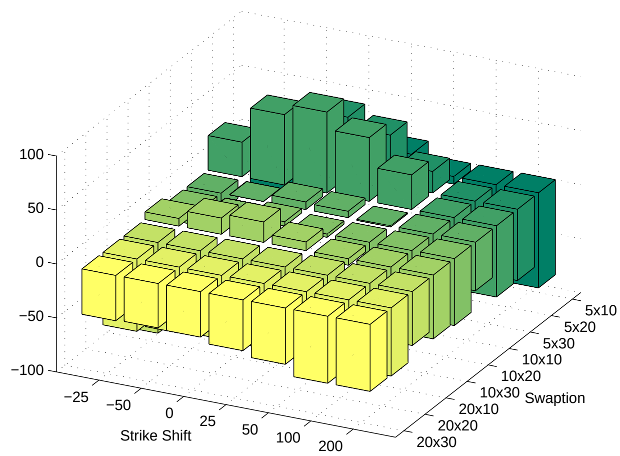

Figure 2: Absolute differences in basis points between market volatilities and those induced by the G211 model.

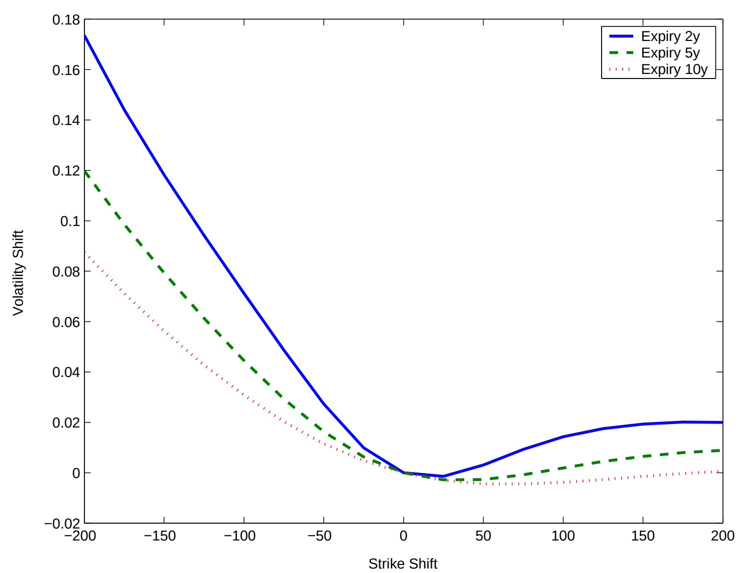

Figure 3: Volatility differences *w.r.t* the corresponding at-the-money values for a ten year swap rate induced by the G211 model, for different expiries.

to incorporate some relevant market information, as, for example, that contained in the right wing of a swaption volatility curve.

The G211 model is calibrated as explained in the previous section, while the used G2 model has parameters that coincide with the initial guess of the first component of the G211 model. In this way, spurious mispricing effects, as those induced by different qualities of calibration to at-the-money values, should be reduced to a minimum.

We price a set of CMS spread options with different maturities, with or without Bermuda-style callability features, applying the general formula (5). We consider, as a relevant case, a swap contract receiving each year the difference between two CMS rates, C_a and C_b, floored at zero and paying, twice a year, the LIBOR rate L plus a spread X. Callability is then added as the option to cancel the contract. The risk-neutral price of this swap is given by:

$$V(t) = E\left[\sum_{i=1}^{n} \tau_i D(t,T_i)\left(C_a(T_{i-1}) - C_b(T_{i-1})\right)^+ - \sum_{i=1}^{n'} \tau_i' D(t,T_i')\left(L(T_{i-1}') + X\right)\right] \quad (6)$$

where $D(t,T_i)$ denotes the (stochastic) discount factor at time t for maturity T_i

$$D(t,T_i) = e^{-\int_0^{T_i} r(t)\,dt},$$

and where T_i's are the payment dates of the exotic leg and τ_i's the associated year fractions, $1 \le i \le n$, while the primed letters denote the corresponding quantities in the LIBOR leg.

We report in Table 10 the prices of three cancellable swaps with different maturities and LIBOR spreads for a = 10y and b = 2y, thus based on the difference, floored at zero, between the ten-year and the two-year swap rates. In this table, we denote by "Intrinsic" the value of the swap without callability features, by "Callability" the value of the Bermudan option to call the swap, and by "Total" the (rounded) sum of the two.

As expected, the three G211 prices, in the three considered cases, are always greater than the corresponding G2 prices. This difference can be explained in terms of the differences in the calibration to swaption smiles between the two models. In fact, as noticed in the previous section, the G211 model can recover market volatilities also for large strikes, as opposed to the G2 model, whose implied volatility curve is always monotonically decreasing. This translates into a larger convexity correction that is implied by the G211 model, and accordingly into a higher G211 "Intrinsic" price, being such correction proportional to underlying swap tenor (larger for the ten-year swap rate than for the two-year swap rate). Moreover, the overall increase in implied volatilities positively affects the Bermudan option price as well, which therefore leads to G211 "Callability" prices that are larger than the corresponding G2 ones.

6 Conclusions

We have proposed a simple interest rate model that is capable of accommodating swaption smiles and CMS convexity corrections. The model is based on a short rate with uncertain parameters, whose random values are drawn at an infinitesimal time.

We then considered the particular case of a short rate model obtained by mixing one two-factor Gaussian model with two one-factor Gaussian models. An effective calibration procedure has been suggested and numerical results have been presented.

Finally, to stress the importance to incorporate the relevant information coming from the market, the pricing of (callable) swaps based on CMS spread options has been compared with that produced by a two-factor Gaussian short rate model.

References

[1] Brigo, D., and Mercurio, F. (2001) Interest Rate Models: Theory and Practice. Springer Finance, Berlin.

[2] Brigo, D., Mercurio, F., and Rapisarda, F. (2004) Smile at the uncertainty. *Risk* 17(5), 97-101.

[3] Gatarek, D. (2003) LIBOR market model with stochastic volatility. Deloitte&Touche. Available online at:
`http://papers.ssrn.com/sol3/papers.cfm?abstract_id=359001`

[4] Hagan, P.S., Kumar, D., Lesniewski, A.S., Woodward, D.E. (2002) Managing Smile Risk. *Wilmott magazine*, September, 84-108.

[5] Hull, J., White, A. (1994a) Numerical Procedures for Implementing Term Structure Models I: Single-Factor Models. *The Journal of Derivatives* 2, 7-16.

[6] Hull, J., White, A. (1994b) Numerical Procedures for Implementing Term Structure Models II: Two-Factor Models. *The Journal of Derivatives* 2, 37-47.

[7] Lourakis, M. (2005) Levenberg-Marquardt non-linear least squares algorithms in `C/C++`. Available online at:
`http://www.ics.forth.gr/~lourakis/levmar`

Expiry	Tenor	Strike -200	-100	-50	-25	25	50	100	200
1y	10y	11.51%	3.24%	1.03%	0.37%	-0.22%	-0.22%	0.21%	2.13%
5y	10y	7.80%	2.63%	1.02%	0.44%	-0.33%	-0.53%	-0.63%	-0.17%
10y	10y	6.39%	2.25%	0.91%	0.40%	-0.31%	-0.52%	-0.71%	-0.47%
20y	10y	5.86%	2.07%	0.85%	0.37%	-0.30%	-0.51%	-0.73%	-0.62%
30y	10y	5.44%	1.92%	0.79%	0.35%	-0.29%	-0.52%	-0.79%	-0.85%
1y	20y	9.45%	2.74%	1.17%	0.46%	-0.24%	-0.25%	0.15%	1.62%
5y	20y	7.43%	2.56%	1.00%	0.43%	-0.32%	-0.51%	-0.60%	-0.10%
10y	20y	6.59%	2.34%	0.94%	0.41%	-0.32%	-0.54%	-0.72%	-0.43%
20y	20y	6.11%	2.19%	0.90%	0.40%	-0.32%	-0.55%	-0.77%	-0.61%
30y	20y	5.46%	1.92%	0.79%	0.35%	-0.29%	-0.50%	-0.72%	-0.69%
1y	30y	9.17%	2.67%	1.19%	0.47%	-0.25%	-0.27%	0.13%	1.58%
5y	30y	7.45%	2.58%	1.01%	0.44%	-0.33%	-0.52%	-0.61%	-0.13%
10y	30y	6.73%	2.38%	0.96%	0.42%	-0.33%	-0.53%	-0.68%	-0.35%
20y	30y	6.20%	2.22%	0.91%	0.40%	-0.32%	-0.54%	-0.74%	-0.55%
30y	30y	5.39%	1.90%	0.78%	0.35%	-0.28%	-0.50%	-0.72%	-0.68%

Table 1: Market volatility smiles for the selected expiry-tenor pairs. Strikes are expressed as absolute differences in basis points *w.r.t* the at-the-money values.

Expiry	Tenor 10y	20y	30y
1y	17.60%	15.30%	14.60%
5y	16.00%	14.80%	14.30%
10y	14.40%	13.60%	13.10%
20y	13.10%	12.10%	11.90%
30y	12.90%	12.30%	12.30%

Table 2: Market at-the-money volatilities.

Maturity	Tenor 10y	20y	30y
5y	94.1	124.1	130.3
10y	82.0	104.8	110.6
15y	72.5	91.3	98.3
20y	66.7	84.2	92.9
30y	64.6	85.2	97.9

Table 3: Market CMS swap spreads in basis points.

	Swap tenors		
Expiry	2 vs 10	2 vs 20	2 vs 30
1y	62.1%	54.7%	52.5%
2y	81.3%	77.2%	75.9%
5y	93.1%	91.4%	90.8%
10y	96.4%	95.5%	95.2%
15y	97.5%	96.8%	96.6%
20y	98.0%	97.4%	97.2%

Table 4: Swap rate correlations implied by market prices of CMS spread options.

G211	**Swaption**			**Swaption**			**Swaption**		
Shift	5x10	5x20	5x30	10x10	10x20	10x30	20x10	20x20	20x30
-50	-42	6	32	-17	-15	7	-84	-67	-42
-25	-1	43	65	0	0	15	-74	-61	-41
0	18	55	75	7	4	19	-65	-59	-42
25	13	46	59	6	-3	8	-61	-60	-47
50	-7	20	32	-1	-12	-6	-59	-63	-52
100	-64	-42	-36	-26	-43	-37	-59	-68	-62
200	-88	-66	-66	-45	-62	-59	-50	-63	-62

Table 5: Absolute differences in basis points between market swaption volatilities (both at-the-money and skew data) and those induced by the G211 model. The shift column lists (in basis points) the value to be added to the at-the-money strike to obtain the option strike.

G211	**Tenor**		
Maturity	10y	20y	30y
5y	0	-1	-2
10y	-2	-3	-5
15y	-2	-4	-7
20y	-3	-5	-9

Table 6: Absolute differences in basis points between market CMS swap spreads and those induced by the G211 model.

	Swap Tenors		
Expiry	2 vs 10	2 vs 20	2 vs 30
1y	1.2%	1.5%	1.5%
2y	1.5%	1.8%	1.9%
5y	0.7%	1.1%	1.5%
10y	0.5%	1.2%	1.9%
15y	0.7%	1.7%	2.8%
20y	0.7%	1.9%	3.1%

Table 7: Absolute differences between input swap rate correlations and those induced by the G211 model.

Date	**Rate**	**Date**	**Rate**	**Date**	**Rate**	**Date**	**Rate**
29-Sep-05	2.12%	19-Dec-07	2.48%	30-Sep-19	3.50%	30-Sep-31	3.79%
03-Oct-05	2.12%	30-Sep-08	2.56%	30-Sep-20	3.54%	30-Sep-32	3.80%
07-Oct-05	2.15%	30-Sep-09	2.67%	30-Sep-21	3.58%	30-Sep-33	3.80%
31-Oct-05	2.15%	30-Sep-10	2.77%	30-Sep-22	3.62%	29-Sep-34	3.80%
30-Nov-05	2.16%	30-Sep-11	2.87%	29-Sep-23	3.65%	28-Sep-35	3.80%
21-Mar-06	2.19%	28-Sep-12	2.97%	30-Sep-24	3.68%	29-Sep-45	3.81%
15-Jun-06	2.23%	30-Sep-13	3.07%	30-Sep-25	3.71%	30-Sep-55	3.80%
21-Sep-06	2.28%	30-Sep-14	3.16%	30-Sep-26	3.73%		
20-Dec-06	2.32%	30-Sep-15	3.24%	30-Sep-27	3.75%		
20-Mar-07	2.36%	30-Sep-16	3.31%	29-Sep-28	3.76%		
21-Jun-07	2.40%	29-Sep-17	3.38%	28-Sep-29	3.78%		
20-Sep-07	2.44%	28-Sep-18	3.44%	30-Sep-30	3.78%		

Table 8: EUR zero-coupon continuously-compounded spot rates (ACT/365).

G211 Shift	Expiry 2y 10y	Expiry 2y 20y	Expiry 2y 30y	Expiry 5y 10y	Expiry 5y 20y	Expiry 5y 30y	Expiry 10y 10y	Expiry 10y 20y	Expiry 10y 30y
-200.00	17.36%	14.18%	13.19%	11.98%	10.68%	10.22%	8.75%	8.53%	8.16%
-175.00	14.41%	11.98%	11.17%	9.86%	8.87%	8.46%	7.12%	6.90%	6.71%
-150.00	11.82%	9.97%	9.33%	7.93%	7.18%	6.92%	5.63%	5.49%	5.28%
-125.00	9.42%	8.05%	7.60%	6.13%	5.59%	5.38%	4.27%	4.20%	4.04%
-100.00	7.12%	6.18%	5.88%	4.45%	4.09%	3.96%	3.08%	2.98%	2.92%
-75.00	4.87%	4.28%	4.10%	2.93%	2.68%	2.60%	2.03%	1.97%	1.88%
-50.00	2.73%	2.42%	2.32%	1.63%	1.49%	1.44%	1.15%	1.13%	1.08%
-25.00	0.99%	0.86%	0.82%	0.63%	0.56%	0.54%	0.48%	0.45%	0.46%
0.00	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
25.00	-0.14%	-0.04%	0.01%	-0.28%	-0.22%	-0.18%	-0.30%	-0.25%	-0.21%
50.00	0.31%	0.48%	0.56%	-0.27%	-0.15%	-0.08%	-0.44%	-0.37%	-0.28%
75.00	0.93%	1.11%	1.22%	-0.08%	0.08%	0.20%	-0.44%	-0.36%	-0.26%
100.00	1.43%	1.61%	1.72%	0.19%	0.37%	0.50%	-0.38%	-0.24%	-0.12%
125.00	1.75%	1.94%	2.04%	0.45%	0.65%	0.80%	-0.27%	-0.12%	0.04%
150.00	1.93%	2.12%	2.23%	0.65%	0.86%	1.01%	-0.14%	0.01%	0.16%
175.00	2.01%	2.21%	2.33%	0.80%	1.02%	1.18%	-0.03%	0.14%	0.31%
200.00	2.00%	2.23%	2.35%	0.89%	1.11%	1.27%	0.06%	0.24%	0.43%

Table 9: Volatility differences *w.r.t* the corresponding at-the-money values induced by the G211 model. The shift column lists (in basis points) the value to be added to the at-the-money strike to obtain the option strike.

	Option Maturity	Libor Spread	Cancellable Swap Intrinsic	Cancellable Swap Callability	Cancellable Swap Total
G2	10y	-250	-119	193	74
G211			-64	214	150
G2	20y	-275	-620	328	-293
G211			-510	396	-114
G2	30y	-300	-559	557	-1
G211			-405	615	210

Table 10: Prices in basis points of cancellable swaps where the difference, floored at zero, between the ten-year and the two-year swap rates is exchanged for LIBOR plus a spread. Prices are calculated with the G2 and G211 models.